Markets with Search Frictions and Partially Informed Intermediary

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Abstract

The paper discusses markets with consumer search frictions and a partially informed intermediary. The intermediary gives consumers individual advice on what products to explore first. The main finding is that with an improvement in the information the intermediary has, the average quality of the product consumers purchase, as well as the total economic welfare and the consumer surplus, might decrease. The mechanism is as follows: if the intermediary gives better average advice to consumers on what product to explore first, all consumers have lower expectations about the next products and explore them less often. That reduces the quality of products purchased by consumers who got wrong advice and might lower the average quality of purchased products. This effect appears in the case of a low search cost, which makes it particularly important to analyze online search intermediaries, such as Google, Amazon, Expedia, etc.

JEL classification: D43, D83, L11, L13, L15.

Keywords: Consumer search, Information, Product heterogeneity, Welfare, Intermediaries, Platforms.

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1 Introduction

In many markets, consumers are initially uninformed about the quality of available products. They may conduct a search to learn about the quality of products, and in many cases, these searches are facilitated by information intermediaries. For example, financial advisors offer information about insurance and investment products, physicians provide advice on appropriate treatments and drugs, and online platforms give consumers a ranking of sellers. As economic activities are increasingly connected through the Internet, consumers can have access to more products at lower search costs, but they also face a much larger set of sellers to choose from. Thus, consumers are increasingly dependent on intermediaries to guide their search (in some deliberate order) for sellers and products. This has led to the enormous commercial success of Internet platforms such as Google, Amazon, and Expedia. In such circumstances, the importance of understanding how intermediaries influence the market increases.

This paper analyzes how the information a platform has about consumer preferences changes market outcomes. There are two main features of the model: first, with some probability, the platform observes the preferences of each consumer individually, and second, it offers each consumer an individual list of firms to visit in some deliberate order. The key variable in the analysis is the probability that the platform knows the quality of various products offered to each consumer. In the special case, where this probability is one, the platform always recommends that the consumer visit the firm with the highest quality products first. The main focus of this paper is a more general case, where this probability is not one. As the probability increases (the platform gets better information about consumer preferences), there are two countervailing forces. The direct effect is that the platform recommends a greater mass of consumers to visit the firm with the highest quality products first, which increases the mean quality of products purchased by consumers. The indirect effect is that consumers expect the next products to be of lower quality, which reduces their incentives to search further and leads to a decrease in the quality of products purchased by those consumers whose preferences the platform does not know.

I demonstrate that in the case of a low search cost, better information leads to a decline in the quality of purchased products, the consumer, and the total economic welfare. However, in the case of a high search cost, the result is the opposite, and better information improves the mean quality of purchased products and welfare.

1.1 Related Literature

The paper is related to the ordered search literature. Arbatskaya (2007) started this branch with a discussion of homogeneous goods. Later Armstrong et al. (2009) and Zhou (2011) investigate ordered search for heterogenous products, in the framework of Wolinsky (1986) and Anderson and Renault (1999). See also recent publications by Parakhonyak and Titova (2018) and Ding and Zhang (2018). In the aforementioned papers, the search ordered is either exogenous or identical for all consumers. A high position in the search order captures the high prominence of a firm. The current paper contributes to the literature by allowing the search order to be consumer-specific (i.e. based on consumer taste). It better captures the business feature in the age of the Internet. Search engines and intermediaries often use “big data” to make an individual recommendation based on consumers’ browsing and search history.

Armstrong (2017), Haan et al. (2018) and Choi et al. (2018) also study models with endogenous ordered search. Prior to search, a consumer receives information about the match quality of the products being sold by every firm. In particular, Choi et al. (2018) show that better information leads to a higher equilibrium price, which is consistent with the finding in my model. However, my paper adds to the literature by showing the effect of the information on the match quality and welfare, demonstrating in particular that the quality of a product purchased by a consumer might be hurt by better information.

The paper is also related to the literature on information intermediaries, which is started by Biglaiser (1993) and Lizzeri (1999). I contribute to their research by introducing heterogeneous products and ordered search settings. A wealth of literature discusses the intermediary as a marketplace that sets fees for firms and ranks them by those fees, but the marketplace itself does not have any information about the quality of their products. Bright representatives of this approach are Athey and Ellison (2011), Chen and He (2011) and Teh and Wright (2018). In these models, firms are sorted by the fees paid, which, in equilibria, is determined by the firms’ heterogeneous quality, known by the firms only, and only in the latter paper, the order of firms is consumer-specific. As a result, consumers explore products in a given order, determined by the platform’s ranking mechanism. I contribute to this branch of literature by discussing the case of a strategic partially informed platform, which autonomously determines the order of search based on incomplete information about consumer preferences.

There is literature on the relation between information and pricing. See Lewis and Sappington (1994), Anderson and Renault (2006) and Anderson and Renault (2000). Recently, Roesler and Szentes (2017) have used the newly developed information design technique a la Kamenica and Gentzkow (2011) and Gentzkow and Kamenica (2016) to further explore this topic. The main message is that more information can be bad because the monopolist can better price discrimination against the consumer. See Boleslavsky et al. (2018) and Armstrong and Zhou (2019) for the effect of information in the competition model, and Dogan and Hu (2018) in a context of consumer search. I contribute to this literature by showing another channel through which more information can hurt the welfare. Richer information helps the platform make a better recommendation to consumers, reducing consumers' incentives to search. As a result, the set of products considered by consumers shrinks, which might lead to a lower quality of purchased products.

The rest of the paper is organized as follows: In section 2, I introduce the model. Later, in section 3, I solve the model and analyze the equilibrium. In section 4, I derive the main results – the effects of the platform having better information on the market, especially on the quality of consumed products, the consumer, and the total welfares. section 5 is a concluding remark.

2 The Model

The economy consists of a monopolistic platform, two firms labeled A and B, and a continuum of consumers with a measure of one. The platform is the only place for firms and consumers to meet. Firms produce horizontally differentiated products incurring a constant marginal cost normalized to zero. The platform steers consumers' search process by providing each consumer an individual order to visit firms, based on the consumers' preferences to firms' products. For each consumer the platform observes his preferences with probability q. With probability $1-q$, the platform does not observe consumer preferences and has to rank products randomly. Consumers do not know whether the platform observes their preferences, but know q. The platform charges firms an *ad valorem* fee proportional to the transaction price. Firms maximize their revenues by setting the price conditional on the platform's ranking mechanism.

As in Wolinsky (1986), a consumer must incur a search cost s to learn the price charged by any particular firm and its product quality. Consumers search sequentially with costless recall. If consumer j buys a product of firm i at price p_i after visiting k firms, he obtains utility

$$U_{j,i} = u_{j,i} - k \cdot s = \epsilon_{j,i} - p_i - k \cdot s$$

where $\epsilon_{j,i}$ is the realization of a random variable with twice differentiable cdf $F(\epsilon)$, pdf

$f(\epsilon)$ and support $[\underline{\epsilon}, \bar{\epsilon}]$. The term $\epsilon_{i,j}$ can be interpreted as the product quality of firm i for consumer j, and is assumed to be independent across consumers and firms. Each consumer can buy one unit of a product at most. The consumer's outside option is low enough to encourage him to purchase the product at any price, which leads to full market coverage.

The market interaction proceeds as follows. All participants know q, the probability that the platform observes the quality $\epsilon_{j,A}$ and $\epsilon_{j,B}$ for a given consumer j. First, firms simultaneously set prices p_A and p_B, conditional on q. After that the platform provides an individual search order for each consumer as follows: if the platform observes the quality of products for a given consumer, it recommends this consumer to visit the firm with a higher quality first; if the platform does not observe the quality, it makes the recommendation at random with equal probabilities. In the equilibrium constructed later, it is optimal for a revenue-maximizing platform to offer consumers such a search order, and for utility optimizing consumers to follow this recommendation. After that, consumers form their expectations about prices and follow the ordered sequential search process with search cost s and costless recall. Thereafter, the consumer buys a utility-maximizing product of the ones he explored and pays the price.

3 Analysis

In this section, I derive the perfect Bayesian equilibrium by means of backward induction. Because of consumers and firms' ex-ante symmetry, I focus on analyzing symmetric equilibrium when firms charge equal prices and get equal demand. First, I derive the consumer optimal search rule and use it to obtain the demand functions and optimal pricing on the market. Thereafter, I show that the price and the revenue of each firm increase with better information (higher q). This verifies that, for a revenue-maximizing platform charging firms an *ad valorem* fee proportional to the price, it is always optimal to use the entire information it has, i.e., if the platform observes the product quality for a given consumer, the platform always recommends him/her to visit the firm with the better product first. Lastly, in the Information Effect on the Market section I use these results to analyze the effects of information on the quality of the product's consumer purchases and the welfare of market participants.

3.1 Consumer Search

For each consumer, the platform observes the quality of products with probability q. The consumer does not know whether the platform observes his preferences. If the platform knows consumer's preferences, it provides the firm with a better product on the first position in the ranking. With probability $1 - q$, the platform does not know consumer's preferences and ranks firms randomly. Hence, each consumer uses the law of total probability, and expects that the first firm in the proposed searching order provides the better product with a probability $q + \frac{1-q}{2} = \frac{1+q}{2}$. Due to a low enough outside option, the consumer always explores the first firm. If both firms charge the same price and the product of the first visited firm has quality z, the net benefit of visiting the second one is derived in Equation 1 as $h(z)$.

$$h(z) = \int_{z}^{\bar{\epsilon}} (\epsilon - z) f_{\epsilon|z}(\epsilon) d\epsilon \ , \tag{1}$$

where $f_{\epsilon|z}(\epsilon)$, described in Lemma 1, is consumer beliefs of the distribution of the second visited firm's product quality, conditional on the observed quality of the product provided by the first firm. In the special case of $q = 0$, the platform does not know anything about consumer preferences and shows products at random. Hence, the quality of the firms' products is uncorrelated, and the consumer simply expects $f_\epsilon(\epsilon)$ to be a distribution of the second firm product quality. In such a case, the model is reduced to Wolinsky (1986) model. If $q > 0$, the platform observes the quality of products for some consumers, and steers them to visit the firm with the better product first. That makes the quality of the second visited firm's product to be correlated with the quality of the one visited first. The higher q is, and the higher z the consumer observes in the first firm, then firmer his belief is that the second product is the worse one, which weakens expectation about the quality of the second product and the incentive to search further.

Lemma 1. *In the region $\epsilon > z$, $f_{\epsilon|z}$ can be expressed as:*

$$f_{\epsilon|z}(\epsilon) = Pr(\epsilon > z|z) \cdot f(\epsilon|z, \epsilon > z) = \frac{\frac{1-q}{2}(1 - F(z))}{\frac{1-q}{2}(1 - F(z)) + \frac{1+q}{2}F(z)} \cdot \frac{f(\epsilon)}{1 - F(z)}$$

The consumer's reservation value x, which solves the equation $s = h(x)$, is the product quality such that the benefit of sampling one more product equals the search cost. If there is no difference in product prices, then the consumer's optimal strategy is to stop searching if the product quality is higher than x. Otherwise, the consumer should explore the second product and purchase the better one. The next two lemmas discuss how x depends on market parameters.

Lemma 2 shows, that in accordance with the classic result of Wolinsky (1986), if the search cost increases, it becomes less profitable to explore the next product. As a result, x decreases.

Lemma 2. *The reservation value x is a decreasing function of search cost s.*

Lemma 3 highlights the idea that if the platform has better information (q increases), consumers have lower incentives to search because the first explored product is the better one with a higher probability.

Lemma 3. *The reservation value x is a decreasing function of q.*

3.2 The Demand

Due to the ex-ante symmetry of firms, they have similar demand functions. Without loss of generality I derive the demand function for firm A. Suppose firm A sets price p_A, and firm B sets the equilibrium price p^*. Define $\Delta = p_A - p^*$. If the consumer visits firm A first, observes the price p_A and expects that the firm B charges the price p^*, he will stop the search and buy if and only if $\epsilon_A - p_A > x - p^*$, or, same, $\epsilon_A > x + \Delta$.

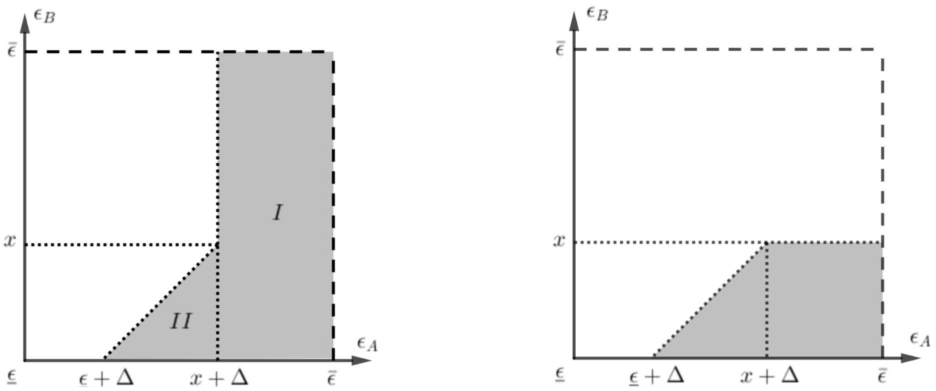
Figure 1 summarizes the demand for firm A. The shaded area in Figure 1a shows firm A's demand if it is visited first. If so, it gets the consumers for whom $\epsilon_A > x+\Delta$ (region I) because these consumers buy from firm A immediately and don't search further. Among the consumers who sample both firms, firm A gets consumers who derive higher utility from consuming its product rather than the product of firm B, i.e. for whom $\epsilon_A > \epsilon_B + \Delta$ (region II). The shaded area in Figure 1b shows the demand for firm A if it is visited second. If so, it gets the consumers for whom $\epsilon_B < x$ and $\epsilon_A > \epsilon_B + \Delta$ because first, these consumers don't stop at firm B and search further and, at second, they value the product of firm A higher than that of firm B. Since each firm is shown first to half of consumers, because ϵ_A, ϵ_B and q are independent, we can express for firm A as:

$$D(p_A, p^*) = \frac{1+F(x)}{2}(1 - F(x+\Delta)) + \int_{\underline{\epsilon}}^{x+\Delta} f(\epsilon)F(\epsilon - \Delta)d\epsilon \tag{2}$$

It's important to note that the demand depends on q only through x.

Figure 1: Demand for firms

(a) Demand if visited first *(b) Demand if visited second*



Assuming that the search cost s is such that $x \in [\underline{\epsilon}, \bar{\epsilon}]$ $\forall q \in [0, 1]$, I now turn to the analysis of the equilibrium. Note that in equilibrium $D(p^*, p^*) = \frac{1}{2}$, i.e. every consumer buys exactly one product.

Assumption 1. *$f(\cdot)$ is a log-concave and continuously differentiable function.*

Bagnoli and Bergstrom (2005) show that under Assumption 1, $F(\cdot)$ and $1 - F(\cdot)$ are also log-concave.

Lemma 4. *Under Assumption 1, there exists a unique symmetric equilibrium:*

$$p_A = p_B = p^* = -\frac{D(p, p^*)}{\frac{\partial D}{\partial p}(p, p^*)}\Big|_{p=p^*} = \frac{1}{(1 - F(x))f(x) + 2\int\limits_{\underline{\epsilon}}^{x} f^2(\epsilon)d\epsilon} \tag{3}$$

As Quint (2014) showed, log-concavity of $f(\epsilon)$, $F(\epsilon)$, and $1 - F(\epsilon)$ guarantees that the demand is log-concave in price, hence, $-\frac{D(p,p^*)}{\frac{\partial D}{\partial p}(p,p^*)}$ is decreasing in p, which guarantees a unique solution of Equation 3.

As shown in Lemma 4, the price is a function of x, which, in turn, is a function of the search cost and the information the platform has. If s increases, then, in accordance with the classic result of Wolinsky (1986), consumers search less often. If q increases, rational consumers expect that the platform makes better ranking of firms and have lower expectations of the second firm's product quality, which weakens the incentives to search further. If consumers search less, the market competitiveness declines and firms can raise prices. These results are summarized in the next lemma.

Lemma 5. *Under Assumption 1, p^* is an increasing function of s and q.*

The equilibrium price is an increasing function of q, while the firms' demand is constant due to a low enough outside option and full market coverage, and hence non-sensitive to the price. That leads to an increase in the firms' profits if the platform uses better information for ranking. Hence, as the platform charges firms an *ad valorem* fee proportional to the price, the platform's revenue is an increasing function of q, which guarantees that it is optimal for the platform to use the entire information it has for ranking, i.e. always recommend the consumer to visit the firm with the better product first if the platform observes the product quality for this consumer.

4 Information Effect on the Market

4.1 Information Effect on the Quality of Purchased Products

In this section, I address the main question of the paper: How would the quality of the product that the consumer purchases vary depending on the information the platform has? On the one hand, as the platform has better information and q increases, the product that the consumer explores first is the better one with a higher probability, which improves the expected quality of the consumed product. On the other hand, according to Lemma 3, as q increases, consumers have lower incentives to search and visit the second firm less often, simply purchasing the first explored product. Hence, consumers, who got a wrong recommendation on which firm to visit first, have smaller chances to choose the better product. That reduces the expected quality of the consumed product. The analysis in this chapter is designed to resolve the ambiguity in the combined effect.

The expected quality of the product, with which the consumer leaves the market, can be found as described in Equation 4, where $P(q)$ is the probability that the consumer purchases the worse product. The first term in the sum stands for the expected quality of the better product, multiplied by the probability the consumer purchases the better product. The second term stands for the expected quality of the worse product, multiplied by the probability the consumer purchases the worse product.

$$V(q) = (1 - P(q)) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; dF_{max\{\epsilon_1,\epsilon_2\}}(\epsilon) + P(q) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; dF_{min\{\epsilon_1,\epsilon_2\}}(\epsilon) =$$

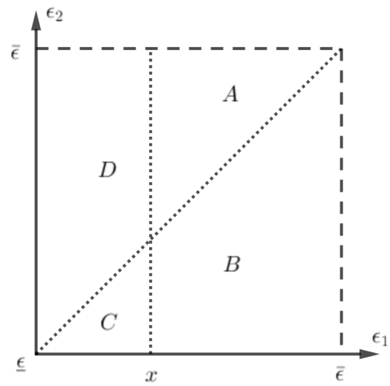
$$= (1 - P(q)) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; d\left[F(\epsilon)^2\right] + P(q) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; d\left[1 - (1 - F(\epsilon))^2\right] \tag{4}$$

Notice that the expected quality of the consumed product is a decreasing function of $P(q)$ as both integrals in Equation 5 are constants and stand for the quality of the better and the worse product among two respectively. In the subsequent analysis, I discuss how

$P(q)$, the probability that the consumer purchases the worse product, varies depending on the information the platform has.

Figure 2 illustrates the probabilities of different search outcomes. ϵ_1 and ϵ_2 stand for the qualities of products of the first and the second firm in consumers' search order, respectively. Regions B and C have combined area $\frac{1+q}{2}$, and illustrate the mass of consumers who find the better product in the first visited firm ($\epsilon_1 > \epsilon_2$). Accordingly, the regions A and D with combined area $\frac{1-q}{2}$ depict the mass of consumers who visit first the firm with the worse product ($\epsilon_1 < \epsilon_2$). In regions B and C, the first visited firm's product is the better one, and a consumer purchases this product even if he visited the second firm. In region D the product quality of the first visited firm is below x; hence the consumer visits the second firm and purchases the better product. In region A, the product quality of the first visited firm is above x; hence the consumer decides do not to search further. But this product is the worse of the two. As a result, the region A is the only one where the consumer leaves the market with the worse product. The probability of that event is represented as the area of the region A in Figure 2 and is indicated in Equation 5.

Figure 2: Probabilities of the search outcomes



$$P(q) = (1-q) \cdot \frac{\left(1-F(x)\right)^2}{2} \tag{5}$$

Differentiating Equation 5 with respect to q, we get:

$$\frac{\partial P(q)}{\partial q} = -\frac{(1-F(x))^2}{2} - (1-q)f(x)(1-F(x))\frac{\partial x}{\partial q} \tag{6}$$

The first term in the sum is the direct effect of increased q, which is always negative and expresses a reduction in the share of consumers who visit the firm with the worse product first. The second term is always positive and stands for the indirect effect explained by a reduction in the consumer search intensity and is associated with the level of this reduction $\frac{\partial x}{\partial q}$. If consumers decrease the search intensity enough in response to increased q, making $\frac{\partial P(q)}{\partial q}$ positive, then the indirect effect of decreased search incentives outweighs the direct effect of a better recommendation, hurting the average quality of purchased products. The condition of that is given in Equation 7.

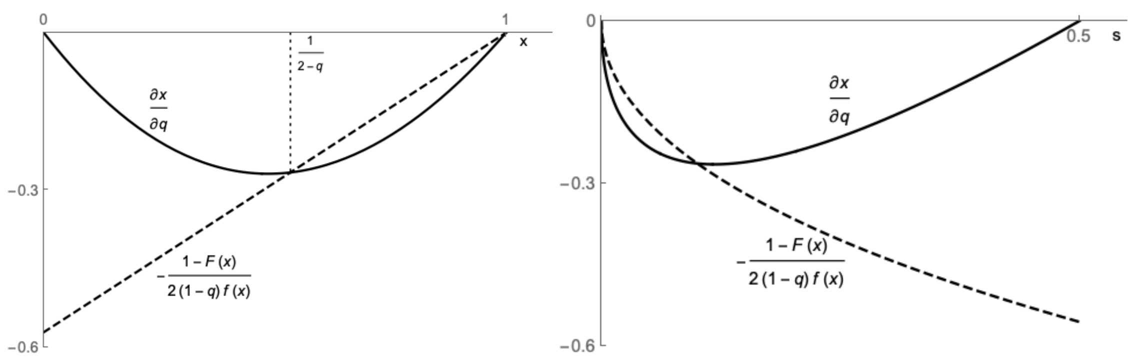
$$\frac{\partial x}{\partial q} < -\frac{1 - F(x)}{2(1-q)f(x)} \tag{7}$$

Figure 3 illustrates the Equation 7 under Assumption 2. The inequality holds for high enough x, or, same, low enough s. Hence, for a low enough search cost, the indirect effect outweighs the direct one, and the average quality of consumed products decreases if the platform has better information.

Assumption 2. *Product quality ϵ is uniformly distributed on $[0, 1]$.*

Figure 3: Illustration of Equation 7. The case of a uniform distribution.

(a) As functions of reservation value x *(b) As functions of search cost s*



Proposition 1 summarizes this result for the general case. The proposition states that, in the case of a high search cost, the expected quality of a purchased product increases with the platform having better information, while in the case of a low enough search cost, it decreases at first and starts to increase thereafter.

Proposition 1. *There exist $\hat{s}$ and $\hat{\hat{s}}$ s.t. $\hat{s} \leq \hat{\hat{s}}$ and*

1. *for any s, s.t. $s > \hat{\hat{s}}$, the expected quality of a consumed product increases in q.*
2. *for any s, s.t. $0 < s < \hat{s}$, there exists $\hat{q}(s)$ s.t.*
   (a) *for any q, s.t. $q > \hat{q}(s)$, the expected quality of a consumed product increases in q.*

(b) *for any q, s.t. $q < \hat{q}(s)$, the expected quality of a consumed product decreases in q.*

the expected quality of a consumed product decreases in q.

Under Assumption 2, $\hat{s} = \hat{\hat{s}}$

The logic behind the proposition is as follows. If a consumer gets a correct recommendation, i.e., visits the firm with the better product first, then, due to firms charge equal prices, he will purchase this product regardless of whether he visits the second firm or not. Therefore, the quality of the product this consumer purchases is unaffected by either the direct or the indirect effect. As a result, both the direct and the indirect effects affect only the consumers, who visit the firm with the worse product first. Further in this paragraph, I focus only on these consumers. Regarding the direct effect, its magnitude increases with the value of the search cost. For a high search cost, or, same, a low x, the direct effect is large because all consumers, who got the product with quality above x, do not visit the second firm. All these consumers are benefiting from an increase in q, because it strengthens the chance they would get a correct recommendation and visit the firm providing a product, which is better for them, first. Now I turn to the discussion of the indirect effect, which is driven by a consumer's beliefs. The consumer does not know whether the platform observes his preferences, hence, he/she is not sure if the product of the first visited firm is the better one. The consumer makes a decision whether to visit the second firm based on the search cost and the expected gain of search, which depends on the consumer's beliefs as to whether the second firm provides the better product, i.e. the platform made a wrong recommendation on what firm to visit first. The consumer uses the quality of the product found in the first visited firm as a signal to estimate the probability that the second one provides the better product. The higher the product quality the consumer finds in the first visited firm, the firmer his belief is that the second firm provides the worse product. The indirect effect affects only the consumers, who, first, got the worse product in the first visited firm, and second, got the product with quality slightly below x, because only these consumers will change their decision not to search and will end up with the worse product as a result of the indirect effect. As mentioned above, the consumer uses the quality of the product he found in the first visited firm to predict the expected gain of visiting the second firm. Hence, if such a consumer observes high product quality in the first visited firm, he believes that this product is the better one with a high probability. As a result, such a consumer dramatically lowers the search intensity in response to an increase in q, which makes the indirect effect large compared to the direct one, which is small for a small search cost as discussed above. As a result, the indirect effect outweighs the direct one in the case of a low search cost. If the search cost is high, x is low, which, as discussed above, makes the direct effect large. Therefore, the direct effect outweighs the indirect one.

4.2 Information Effect on Welfare

Due to a sufficiently low outside option, all consumers search at least once, and search the second time only if the quality of the first product is below x. The first explored product is the better one for $\frac{1+q}{2}$ portion of consumers, while for the remaining mass of consumers $\frac{1-q}{2}$, the first product is the worse one. As a result, the level of the consumer's search expenditures can be expressed as shown in Equation 8.

$$\begin{aligned} SE(q) &= s \cdot \Big(1 + \frac{1+q}{2} \cdot F_{max\{\epsilon_1,\epsilon_2\}}(x) + \frac{1-q}{2} \cdot F_{min\{\epsilon_1,\epsilon_2\}}(x)\Big) \\ &= s \cdot \Big(1 + \frac{1+q}{2} \cdot F(x)^2 + \frac{1-q}{2} \cdot \big(1 - (1 - F(x))^2\big)\Big) \end{aligned} \tag{8}$$

When the platform has better information about consumer preferences, there are two effects. First, there is a higher probability that consumers get the better product at the first visited firm. Second, consumers expect lower quality of the second product. Both effects lower consumers' incentives to search. This result is summarized in the next lemma.

Lemma 6. *Search expenditures are a decreasing function of q.*

Define the Total Surplus in Equation 9 as

$$TS(q) = V(q) - SE(q) \tag{9}$$

Lemma 6 shows that $SE(q)$ is decreasing in q. Proposition 1 gives the condition, under which $V(q)$ is increasing and decreasing in q. Hence the net effect may be ambiguous. Proposition 2 provides conditions under which the Total Surplus is decreasing and increasing in q.

Proposition 2. *For any $q \in [0, 1)$, there exist $\check{s}$ and $\check{\check{s}}$ s.t. $\check{s} \leq \check{\check{s}}$ and*

1. *for any s, s.t. $0 < s < \check{s}$, the total economic welfare locally decreases in q.*

2. *for any s, s.t. $s > \check{\check{s}}$, the total economic welfare locally increases in q.*

Under Assumption 2, $\check{s} = \check{\check{s}}$

For a low enough search cost, as q increases, the savings in search expenditures are relatively small and outweighed by the reduction in the quality of the product that the consumer purchases. If the search cost is above $\hat{s}$, defined in Proposition 1, the quality of the purchased product increases with q, while the search expenditures decreases, hence, both effects increase the Total Surplus. In the intermediate case $\check{s} < s < \check{\check{s}}$ the savings in the search expenditures are comparatively high and outweigh the reduction in the quality of the product that the consumer purchases.

According to Lemma 5, the price increases in q, while, according to Proposition 2, for a low enough search cost, the Total Surplus decreases in q. Hence, if s is low, the Consumer Surplus, defined in Equation 10, decreases in q. However. for a high enough search cost, the Total Surplus is an increasing function of q. Therefore, the effect of increased q on the Consumer Surplus is ambiguous in the general case and depends on the levels of s and q. This result is summarized in Proposition 3.

$$CS(q) = V(q) - SE(q) - p^*(q) \tag{10}$$

Proposition 3. *For any $q \in [0, 1)$, there exists $\tilde{s}$ s.t. for any $q \in [0, 1)$, there exists $\tilde{s}$ and $\tilde{\tilde{s}}$ s.t. $\tilde{s} \leq \tilde{\tilde{s}}$ and*

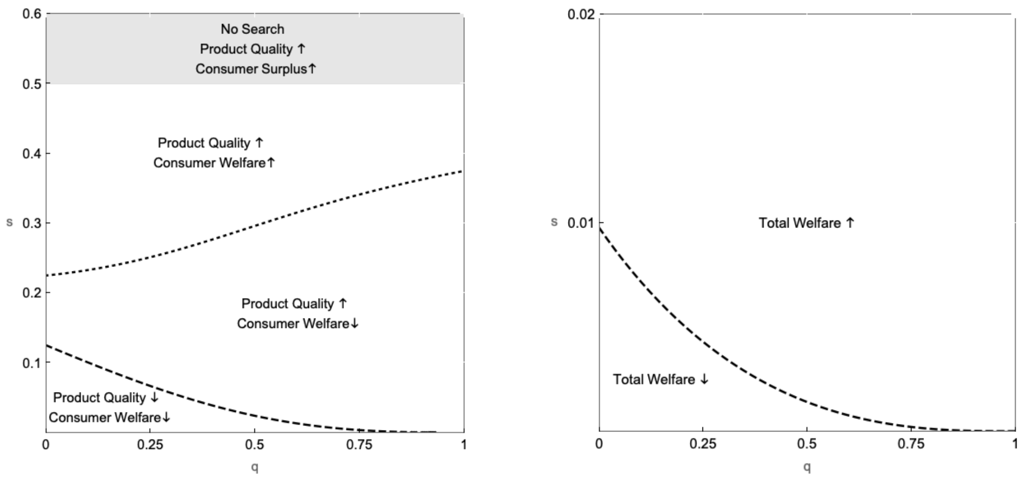
1. *for any s, s.t. $0 < s < \tilde{s}$, the consumer welfare locally decreases in q.*
2. *for any s, s.t. $s > \tilde{\tilde{s}}$, the consumer welfare locally increases in q.*

Under Assumption 2, $\tilde{s} = \tilde{\tilde{s}}$

Figure 4 illustrates the compositions of s and q which lead to different changes of the market outcomes in case of uniform distribution ϵ. If $s > \frac{1}{2}$, then $x < 0 = \underline{\epsilon}$, hence, consumers never explore the second product. Therefore, if q increases, the price does not change and the Consumer Surplus increases, as does the Total Surplus. If $0 < s < \frac{1}{2}$ and $0 < q < 1$, then $0 < x < 1$ and the consumers who explored the product with quality below x, search the second product as well. Consistent with Proposition 1, for low enough search cost s, the expected quality of the consumed product is decreasing with q. Moreover, according to Figure 4a, the Consumer Surplus increases with q for a high enough search cost because the effects of increased quality of the consumed product and decreased search expenditures outweigh the increase in the price.

Figure 4: The effect of levels of search costs s and platform's information q on market outcomes. The case of uniform distribution.

(a) The effect on the product quality and consumers' welfare *(b) The effect on the total economic welfare*



5 Concluding Remark

The paper discusses markets with consumer search frictions and a partially informed intermediary. The main finding is that, with an improvement in the information the intermediary has about consumer preferences, the average quality of the product consumers purchase might decrease. The intuition behind the mechanism is as follows: if the intermediary has better information and gives better advice to consumers on what product to explore first, consumers have lower expectations about the quality of the next products and explore them less often, which reduces the number of explored products and might lower the quality of the one chosen. I also show that consumers and the whole economy can benefit or be hurt if the intermediary has more information about preferences and can better manipulate the order, in which consumers explore products. The actual effect depends on the search costs. In the case of a low enough search cost, the consumer welfare and the total welfare decrease if the platform has better information, while in the case of a high enough search cost, both types of welfare increase.

One of the possible extensions of the model for further research is to reduce the monopoly power of the intermediary, for example, by introducing competition between intermediaries. That will force the intermediary to incorporate the consumer surplus in its objective function. It is particularly important that, in this case, the intermediary may prefer not to use all the information it has about consumer preferences and to steer the search order to keep the incentives for the firm not to raise prices too much.

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Appendices

Proof. of Lemma 1

Suppose that ϵ_1, ϵ_2 and ξ are mutually independent, where $Pr(\xi = 1) = 1 - Pr(\xi = 0) = \frac{1-q}{2}$ and ϵ_1, ϵ_2 are identically distributed with density $f(\epsilon)$. Let $V = min\{\epsilon_1, \epsilon_2\}$ and $W = max\{\epsilon_1, \epsilon_2\}$ and define $X = \xi \cdot V + (1-\xi) \cdot W$, $Y = (1-\xi) \cdot V + \xi \cdot W$. I seek for $f_{Y|X}(y|x)$ on the region where $y > x$.

Step 1. Joint density for (V, W)

The joint cdf for (V, W) for $w \geq v$ given by:

$$\begin{aligned} F_{V,W}(v,w) &= Pr(V \leq v, W \leq w) \\ &= Pr(\epsilon_1 \leq v, v < \epsilon_2 \leq w) + Pr(v < \epsilon_1 \leq w, \epsilon_2 \leq v) + Pr(\epsilon_1 \leq v, \epsilon_2 \leq v) = \\ &= 2F(v)[F(u) - F(v)] + F(v)^2 \end{aligned}$$

So the density is given by:

$$f_{V,W}(v,w) = \frac{d^2 F_{V,W}(v,w)}{dvdw} = 2f(v)f(w), \quad for\ v \leq w$$

Step 2. Joint density for (X, Y)

X and Y are functions of (V, W, ξ), so I first derive the density for $(X, Y, \xi) = g(V, W, \xi)$. It follows that $g^{-1}(x, y, \xi) = (\xi x + (1-\xi)y, (1-\xi)x + \xi y, q)$ with Jacobian $J(x,y,\xi) = \begin{pmatrix} \xi & 1-\xi & x-y \\ 1-\xi & \xi & y-x \\ 0 & 0 & 1 \end{pmatrix}$,

so $|detJ(x,y,\xi)| = |\xi^2 - (1-\xi)^2| \cdot 1 = |\xi - (1-\xi)| = |1 - 2\xi| = 1$

Hence the density for (X, Y, ξ) is given by

$f_{X,Y,\xi}(x,y,\xi) = f_{V,W}(\xi x + (1-\xi)y, (1-\xi)x + \xi y) \cdot f_\xi(\xi)$

where I used that ξ is independent of (ϵ_1, ϵ_2) and therefore also independent of (V, W). Next I obtain the density for (X, Y) by integrating out ξ.

$f_{X,Y}(x,y) = \frac{1-q}{2} f_{V,W}(x,y) + \frac{1+q}{2} f_{V,W}(y,x)$.

It follows that $f_{X,Y}(x,y)\mathbb{I}_{x<y} = \frac{1-q}{2} f_{V,W}(x,y)\mathbb{I}_{x<y}$ because $f_{V,W}(y,x) = 0$ for $x < y$.

Step 3. Marginal density for X.

The density for X is given by:

$$f_X(x) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} f_{X,Y}(x,y) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\frac{1-q}{2} f_{V,W}(x,y) + \frac{1+q}{2} f_{V,W}(y,x) \right) dy =$$
$$= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\frac{1-q}{2} 2f(x)f(y)\mathbb{I}_{x\leq y} + \frac{1+q}{2} 2f(y)f(x)\mathbb{I}_{x>y} \right) dy =$$
$$= 2f(x) \left(\frac{1-q}{2} \int_{x}^{\bar{\epsilon}} f(y)dy + \frac{1+q}{2} \int_{\underline{\epsilon}}^{x} f(y)dy \right) =$$
$$= 2f(x) \left(\frac{1-q}{2} \big(1 - F(x)\big) + \frac{1+q}{2} F(x) \right)$$

Step 4. Conditional density of Y given X.

Finally for $y > x$ we have

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{\frac{1-q}{2} f_{V,W}(x,y)}{2f(x)\left(\frac{1-q}{2}\big(1-F(x)\big) + \frac{1+q}{2}F(x)\right)} =$$
$$= \frac{\frac{1-q}{2} 2f(x)f(y)}{2f(x)\left(\frac{1-q}{2}\big(1-F(x)\big) + \frac{1+q}{2}F(x)\right)} = \frac{\frac{1-q}{2}(1-F(x))}{\frac{1-q}{2}(1-F(x)) + \frac{1+q}{2}F(x)} \cdot \frac{f(y)}{1-F(x)}$$

Q.E.D.

Proof. of Lemma 2

From Equation 1 and definition of x we have:

$$s = \int_{x}^{\bar{\epsilon}} (\epsilon - x) f_{\epsilon|x}(\epsilon) d\epsilon \Rightarrow s = \frac{\frac{1-q}{2}(1-F(x))}{\frac{1-q}{2}(1-F(x)) + \frac{1+q}{2}F(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) \frac{f(\epsilon)}{1-F(x)} d\epsilon \Rightarrow$$
$$s = \frac{1-q}{1-q+2qF(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon \Rightarrow$$
$$1 = \frac{\partial}{\partial s} \left(\frac{1-q}{1-q+2qF(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon \right) \Rightarrow$$
$$1 = -\frac{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x) \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon)\, d\epsilon \right)}{(1-q+2qF(x))^2} \cdot \frac{\partial x}{\partial s} \Rightarrow$$

$$\frac{\partial x}{\partial s} = -\frac{(1-q+2qF(x))^2}{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon\right)} < 0 \ \forall\ (q,x) \in [0,1) \times [\underline{\epsilon}, \bar{\epsilon})$$

The alternative expression is $\frac{\partial x}{\partial s} = -\frac{1-q+2qF(x)}{2qsf(x)+(1-q)(1-F(x))}$ Q.E.D.

Proof. *of Lemma* 3

From Equation 1 and definition of x we have:

$$s = \int\limits_x^{\bar{\epsilon}}(\epsilon - x)f_{\epsilon|x}(\epsilon)d\epsilon \Rightarrow s = \frac{\frac{1-q}{2}(1-F(x))}{\frac{1-q}{2}(1-F(x)) + \frac{1+q}{2}F(x)} \cdot \int\limits_x^{\bar{\epsilon}}(\epsilon - x)\frac{f(\epsilon)}{1-F(x)}d\epsilon \Rightarrow$$

$$s = \frac{1-q}{1-q+2qF(x)} \cdot \int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon \Rightarrow$$

$$0 = \frac{\partial}{\partial q}\left(\frac{1-q}{1-q+2qF(x)} \cdot \int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon\right) \Rightarrow$$

$$0 = -\frac{2F(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon}{(1-q+2qF(x))^2}$$

$$0 = -\frac{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon\right)}{(1-q+2qF(x))^2} \cdot \frac{\partial x}{\partial q} \Rightarrow$$

$$\frac{\partial x}{\partial q} = -\frac{2F(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon}{(1-q)\left((1-F(x))\,(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon\right)} < 0 \ \forall\ (q,x) \in [0,1) \times [\underline{\epsilon}, \bar{\epsilon})$$

The alternative expression is $\frac{\partial x}{\partial q} = -\frac{2sF(x)}{(1-q)(2qsf(x)+(1-q)(1-F(x)))}$ Q.E.D.

Proof. of Lemma 5

After differentiating Equation 3 and accounting for log-concavity of $1-F(x)$, we have:

$$\frac{\partial p^*}{\partial q} = -\frac{f(x)^2 + (1-F(x))f'(x)}{\left((1-F(x))f(x) + 2\int\limits_{\underline{\epsilon}}^{x} f(\epsilon)^2 d\epsilon\right)^2} = -\frac{\left((1-F(x))'\right)^2 + \big(1-F(x)\big)\big(1-F(x)\big)''}{\left((1-F(x))f(x) + 2\int\limits_{\underline{\epsilon}}^{x} f(\epsilon)^2 d\epsilon\right)^2} < 0$$

Q.E.D.

Proof. of Proposition 1

Equation 6 can be rewritten as:

$$\Lambda(x,q) := -\frac{2(1-q)f(x)\frac{\partial x}{\partial q}}{1-F(x)} = 1$$

$$\Lambda(x,q) = \frac{4f(x)F(x)\int\limits_{x}^{\bar{\epsilon}} f(\epsilon)(\epsilon - x)\,d\epsilon}{(1-F(x))\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int\limits_{x}^{\bar{\epsilon}}(\epsilon - x)F'(\epsilon)d\epsilon\right)} = 1 \tag{11}$$

$\Lambda(x,q)$ is continuous function of x. $\Lambda(0,q) = 0,\ \lim\limits_{x\to\hat{\epsilon}} \Lambda(x,q) = \frac{2}{1+q} > 1\ \forall q \in [0,1)$. Hence,
$\exists \hat{s}\ s.t.\ \Lambda(x(\hat{s}),q) = 0,\ \forall s > \hat{s}:\ \Lambda(x(s),q) > 0$ and
$\exists \hat{\hat{s}}\ s.t.\ \Lambda(x(\hat{\hat{s}}),q) = 0,\ \forall s > \hat{\hat{s}}:\ \Lambda(x(s),q) < 0$

Under Assumption 2, Equation 6 can be expressed as:

$$\frac{\partial P(q)}{\partial q} = \frac{(1-x)^2((2-q)x-1)}{2(qx+1)},$$

which is positive if $x > \frac{1}{2-q}$ (low s) and negative if $x < \frac{1}{2-q}$ (high s).

x is a continuous and decreasing function of s. Also, if $s = \frac{1}{2}$, then $x = 0$, and if $s = 0$, then $x = 1$. Hence, for any $q \in [0,1)$ we always can chose s such that makes x equals any desired number between 0 and 1.

As a result, for any $\hat{q} \in [0,1)$, exists $\hat{s}$ such that $x(\hat{s}) = \frac{1}{2-\hat{q}}$. For any $s > \hat{s}$, $x(s) < x(\hat{s})$ and $\frac{\partial P(q)}{\partial q}$ is negative. While for any $s < \hat{s}$, $x(s) > x(\hat{s})$ and $\frac{\partial P(q)}{\partial q}$ is positive. Q.E.D.

Proof. of Lemma 6

After differentiating Equation 8 we have:

$$\frac{\partial SE}{\partial q} = \underbrace{-s(1-F(x))F(x)}_{<0} + \underbrace{f(x)(1-q+2qF(x))}_{>0} \cdot \underbrace{\frac{\partial x}{\partial q}}_{<0} < 0$$

Q.E.D.

Proof. of Proposition 2

Under Assumption 2, Equation 9 can be expressed as:

$$TS(q) = \frac{-q^2\left(-3x^2+x+1\right)(1-x)^2 + q\left(-3x^4+10x^3-6x^2+2x+1\right) + x(x+1)(5-3x)}{6(1-q(1-2x))}$$

Differentiating this expression with respect to q, accounting to the fact that x is a function of q, we get:

$$\frac{\partial TS(q)}{\partial q} = \frac{(1-x)^2\left(-\left(q^2+q-4\right)x - 3(3-q)qx^3 - (6-(11-q)q)x^2 - q + 1\right)}{6(qx+1)(1-q(1-2x))},$$

which is negative if and only if $x > \frac{2+\sqrt{10}}{6}$ and $q > \check{q} = \frac{(1-x)\left(-9x^2+2x+1\right)+\sqrt{81x^6-126x^5+67x^4-24x^3-x^2+6x+1}}{2x(3x^2-x-1)}$. Where $\check{q}$ is positive and monotonically increasing if $x > \frac{2+\sqrt{10}}{6}$ and equals zero if $x = \frac{2+\sqrt{10}}{6}$ and equals 1 if $x = 1$.

Hence, for any $q \in [0,1)$, there is exist $\check{s}$ s.t. $q = \check{q}$. For any $s < \check{s}$, $q > \check{q}$, resulting in $\frac{\partial TS(q)}{\partial q} < 0$. For any $s > \check{s}$, $q < \check{q}$, and $\frac{\partial TS(q)}{\partial q} > 0$

Fix any $q \in [0,1)$. There is exist $\check{s}$ when x is such that $q = \check{q}$. For any $s < \check{s}$, $q > \check{q}$, resulting in $\frac{\partial TS(q)}{\partial q} < 0$. For any $s > \check{s}$, $q < \check{q}$, and $\frac{\partial TS(q)}{\partial q} > 0$ Q.E.D.

Proof. of Proposition 3.

According to Proposition 2, Total Surplus decreases in q for low enough search cost, while, Lemma 5 states that price always increases in q. Therefore, Consumer Surplus, defined in Equation 10 as the difference of Total Surplus and price, decreases for low enough search cost, which means $\forall q \in [0,1)$, $\exists \tilde{s}$ s.t. $\forall s < \tilde{s}$: Consumer Surplus decreases in q.

Plug in Equation 10 $V(q)$, $SE(q)$ and $p^*(q)$, defined in Equation 4, Equation 8 and Equation 3 respectively and differentiate with respect to q we get:

$$\begin{aligned}\frac{\partial CS}{\partial q} &= (1-F(x))f(\epsilon)\left(1-F(x)+2(1-q)F(x)\frac{\partial x}{\partial q}\right)\int\limits_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon f(\epsilon)(2F(\epsilon)-1)d\epsilon + \\ &+ s\left(F(x)(1-F(x)) - (1-q+2qF(x))f(x)\frac{\partial x}{\partial q}\right) + \frac{\frac{\partial x}{\partial q}\left(f(x)^2+(1-F(x))f'(x)\right)}{\left(2\int\limits_{\underline{\epsilon}}^{x} f(\epsilon)^2\,d\epsilon + (1-F(x))f(x)\right)^2}\end{aligned} \tag{12}$$

The expression above is continuous function of x.

Plug expression for $\frac{\partial x}{\partial q}$, found in Proof of Proposition 3 and plug $s = \int\limits_{z}^{\bar{\epsilon}}(\epsilon - z)f_{\epsilon|z}(\epsilon)d\epsilon = \frac{(1-q)}{2qF(x)-q+1}\int\limits_{x}^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon$ and estimate $\frac{\partial CS}{\partial q}$ at point $x = \underline{\epsilon}$, we get:

$$\left.\frac{\partial CS}{\partial q}\right|_{x=\underline{\epsilon}} = \int\limits_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon f(\epsilon)(2F(\epsilon)-1)\,d\epsilon > 0 \tag{13}$$

Hence, due to $\frac{\partial CS}{\partial q}$ is continuous in x, $\forall q \in [0,1)$, $\exists \tilde{\tilde{s}}$ s.t. $\forall s > \tilde{\tilde{s}}$: Consumer Surplus increases in q.

Under Assumption 2, differentiating Equation 10 with respect to q and plugging in $s = \int\limits_x^{\bar{\epsilon}} (\epsilon - x) f_{\epsilon|x}(\epsilon) d\epsilon = \frac{(1-q)(1-x)^2}{2(1-q(1-2x))}$, we get

$$\frac{\partial CS}{\partial q} = \frac{1-x}{6(1-q)(x+1)^2(qx+1)(2qx-q+1)} \cdot \lambda(x,q),$$

where

$$\begin{aligned}\lambda(x,q) =& \big[3q(1-q)(3-q)x^6 + 2(1-q)\left(3-q-q^2\right)x^5 + (1-q)\left(2-19q+5q^2\right)x^4 - \\ &- (1-q)\left(11-4q-3q^2\right)x^3 - \left(3-2q+13q^2-2q^3\right)x^2 - (1-q)(1+q)^2 x + (1-q)^2\big]\end{aligned}$$

The first, fractional multiplier is non-negative $\forall (q,x) \in [0,1) \times [0,1]$. Next I show that $\lambda(x,q)$ is negative for high x (low s), positive for low x (high s) and strictly monotone decreasing in x (increasing in s), which will prove the proposition. First, $\lambda(0,q) = (1-q)^2 > 0$ and $\lambda(1,q) = -6(1+q) < 0$. Second, $\frac{\partial \lambda}{\partial x} < 0 \;\; \forall (q,x) \in [0,1) \times [0,1]$. Hence $\forall \tilde{q} \; \exists \tilde{s}$ s.t. $\lambda(x(\tilde{s}), \tilde{q}) = 0$ and $\forall s < \tilde{s}$, $\lambda(x(s), \tilde{q}) < 0$, making $\frac{\partial CS}{\partial q}$ negative. While $\forall s > \tilde{s}$, $\lambda(x(s), \tilde{q}) > 0$, making $\frac{\partial CS}{\partial q}$ positive.

Q.E.D.